Peak Water Demand Study

Probability Estimates for Efficient Fixtures in Single and Multi-family Residential Buildings

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January 2017

Acknowledgments

This research effort could not have been possible without the financial sponsorship provided by the International Association of Plumbing and Mechanical Officials (IAPMO), the American Society of Plumbing Engineers (ASPE), and the Water Quality Association (WQA) for the acquisition of a specially designed end uses of water database from Aquacraft, Inc. for the determination of fixture use probabilities and flow rates.

The research team wishes to express sincere gratitude for the additional sponsorship from the above organizations who awarded a fellowship grant for Toritseju Omaghomi who has been instrumental in querying the statistical database and providing analysis for the research team.

The first draft edition of the Peak Water Demand Study was peer reviewed by a select group of specialists having an academic background in statistical theory and acquainted with Hunter's curve for predicting water supply demand loads. Their comments and suggestions were valuable in steering this study toward the final conclusion. The following individuals participated in the peer review.

- C.J. Lagan, Senior Manager, Testing & Compliance, LIXIL Water Technology, American Standard, U.S.
- E.J.M. (Mirjam) Blokker, PhD MSc, Principal Scientist – Water Infrastructure, KWR Watercycle Research Institute, the Netherlands
- Eric Yeggy, Director of Technical Affairs, Water Quality Association, U.S.
- Jim Lutz, Researcher, Hot Water Research, U.S.
- Gary Klein, Principal, Gary Klein Associates, U.S.

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B	Bathtub
C	Clothes washer
D	Dishwasher
E[*x*]	Expected value of *x*
F	Faucet
gpm	Gallon per minute, L^3/T
H(n,p)	Dimensionless Hunter's Number
k	Fixture group
K	Number of Identical fixtures
L	Lavatory faucet
n	Number of fixture
p	Probability of fixture use
psi	Pounds per square inch
P_0	Probability of zero busy fixtures
q	Fixture flow rate
$Q_{0.99}$	99th percentile of demand flow, L^3/T
S	Shower
T	Toilet
x	Number of busy fixtures
$Z_{0.99}$	99th percentile of the standard normal distribution

Greek

Σ	Summation operation

Executive Summary

1.0 Introduction

In July 2011, the International Association of Plumbing and Mechanical Officials (IAPMO) and the American Society of Plumbing Engineers (ASPE) convened a special task force to revise the methodology for properly estimating premise water supply demands in response to the increased use of water-conserving plumbing fixtures, fixture fittings and appliances and the subsequent decreased demand for water in commercial buildings and residences. In collaboration with IAPMO and ASPE, the Water Quality Research Foundation (WQRF) became a co-sponsor in funding the research project. The Pipe Sizing Task Group was organized with three members from ASPE who specialized in statistics and mathematics (later reduced to two members). An additional member from the University of Cincinnati joined this effort.

The Pipe Sizing Task Group recognized that acquisition and analysis of high resolution water use data was a vital first step in the investigation to develop a probability model for predicting peak water demands in buildings. The task group initiated talks with Aquacraft, Inc. to access the largest U.S. database containing residential end uses of water surveys (REUWS). With sponsorship, the task group contracted a specially designed water use database containing parameters to determine fixture use probabilities and flow rates. Because the dataset provided statistics for residential end use only, the scope of work was narrowed to single and multi-family residential dwellings.

Since the water supply flow rate to plumbing fixtures is today significantly less than the flows used in developing the original Hunter's curve, the scope of the work was further narrowed to determine fixture use probabilities and flow rates only for indoor water-conserving plumbing fixtures. Residential indoor efficient fixtures considered in the database were toilets, showers, dishwashers, clothes washers, and faucets (kitchen and lavatory). Bathtubs were also included but are not considered water-conserving since there are no design benefits to low-flow tub spouts.

The charge of the task group was to develop a statistically based probability model that would predict the peak water demand for single and multi-family dwellings having water-conserving plumbing fixtures. This model would be used to predict the peak water demand for the building supply and principal branches and risers of a residential plumbing system.

2.0 Water Use Database

2.1 *National Survey*

The database consists of water use measurements taken between 1996 and 2011 at over 1,000 single-family homes across the United States. The water use data were recorded with a portable data logger connected to the main water supply pipe in each home. The data logger recorded the volume of water flowing through the main pipe every 10 seconds. The recorded flows were analyzed by Aquacraft using their proprietary Trace Wizard© software and disaggregated into individual water use events. Each water use was associated with one of three mutually exclusive household draws: an indoor fixture, an outdoor fixture or a leak. Only indoor fixture use is considered in this study.

To facilitate queries, the water use data are stored in MS Access format. As shown in Table 1, the database contains two types of information, namely (i) household survey data and (ii) measured flow data. The household survey data reflect characteristics of the home, the residents and the water fixtures; the measure flow data describe the duration, volume, and number of water use events at each fixture group identified in the home. The maximum data-logging period at each home was 14 days. To capture the diurnal variation in indoor residential water use, results for each fixture in each home were summarized on an hourly basis.

Table 1 Survey and measured water use data in the MS Access database.

Household Survey Data	Measured Flow Data
- Number of residents and age distribution	- Number of times a fixture was used
- Type of fixtures	- Duration of each fixture use event
- Number of each fixture type/group	- Volume of each fixture use event
- Number of renovated or retrofitted fixtures	- Daily observed fixture peak flow
- Number of bedrooms and bathrooms	- Logging dates

The national distribution of 1058 surveyed households is summarized in Table 2. A small percentage (2%) of homes was dropped from the analysis due to vacations or other conditions that gave zero or minimal water use. The remaining 1038 homes had a total of 2,821 occupants who generated nearly 863,000 water use events during 11,385 home-days of monitoring. On a per household basis, this translate to an average of 11 trace days per home, 2.72 residents per home and 831.4 water use events per home. Figure 1 illustrates the geographic location of the 1038 homes that participated in the national water use survey conducted by Aquacraft during the period 1996 to 2011.

Table 2 Homes surveyed.

Location by State	Number of Homes	Number of Occupants	Survey Years
Arizona, AZ	17	41	2007 - 2009
California, CA	447	1326	2006 - 2009
Colorado, CO	206	533	1996, 2007 - 2010
Florida, FL	32	78	2007 - 2009
Kentucky, KY	58	128	2007
Nevada, NV	20	593	2007 - 2009
New Mexico, NM	237	44	2010 - 2011
Oregon, OR	24	66	2007 - 2009
Utah, UT	17	56	2007 - 2009
Responded to Survey	1058	2865	-
Invalidated Homes	(20)	(44)	-
Total Analyzed	1038	2821	-

Figure 1 Homes from 62 cities in nine States participated in the national water use survey.

2.2 *Water Use Data*

Six unique fixtures were common to most of the 1038 participating homes (see Table 3). The database does not differentiate between kitchen faucets and lavatory faucets; therefore, both are included in the fixture group “faucet”. Water use falling outside these six categories was lumped into “Other”. For instance, some homes had evaporative coolers, water treatment devices, or unknown fixtures. In addition, leaks were common. The total volume of water used at each fixture differs due to the fixture function and frequency of use. Table 4 is a breakdown of water use per capita in 1038 households. Toilets had the highest use as gallons per capita daily (GPCD), while dishwashers had the lowest use. The mean daily water use was 60.10 GPCD. Nearly 98% of the homes registered a leak. Although leakage accounted for nearly 17% of the volume of daily water use, leaks are not a design factor and, hence, are not considered further. Individual fixture water use as a percentage of total use is shown in Figure 2.

Table 3 Six fixture groups were common to most homes.

Fixture Group	**Abbreviation**	**Number of Homes w/ Group**	**Number of Fixtures in Group**	**Average Fixtures per Home**
Bathtub	B	519	852	1.64
Clothes Washer	C	1002	1002	1.00
Dishwasher	D	722	728	1.01
Faucets	F	1038	4013	3.87
Shower	S	1014	2132	2.10
Toilet	T	1037	2502	2.41

Table 4 Frequency and volume of water use at 1038 single family homes.

Fixture	**Water use events (per capita per day)**	**Volume (GPCD)**
Bathtub	0.08	1.54
Clothes Washer	0.97	14.31
Dishwasher	0.33	0.77
Faucet	22.74	11.87
Shower	0.76	12.59
Toilet	5.80	15.18
Others	9.74	3.84
Leaks	50.11	11.98
Totals (excluding leaks)	40.42	60.10

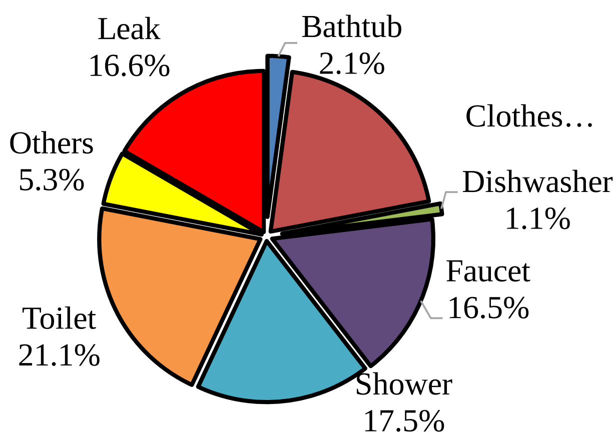

Figure 2 Daily per capita water use at fixtures at 1038 residential households.

3.0 Estimating Fixture Parameters

There are three key parameters in most models formulated to predict peak residential water demand, namely:

- n = number of fixtures in the dwelling
- p = probability that a fixture is in use during the peak period
- q = flow rate at a busy fixture

The fixture count, n, is straight-forward. It is simply the number of indoor fixtures at the dwelling. The fixture count is easily determined directly from the construction plans. In contrast, the p-values and q-values are more elusive. Both must be estimated from observations of fixture use at residential buildings.

3.1 *p-values*

The probability of fixture use is estimated as a dimensionless ratio

$$p = \frac{duration\ of\ \ time\ that\ the\ \ fixture\ is\ busy\ (i.e.,\ running\ \ water)}{duration\ of\ \ time\ that\ the\ \ fixture\ is\ observed}$$

The "duration" terms needed in the numerator and the denominator were obtained for each fixture group from a careful analysis of the national database of high resolution water use recorded at over 1,000 single family homes. Residential water use follows a strong diurnal pattern. This implies that estimates of p will vary from hour to hour. For purposes of estimating peak demand, the critical period of observation is the hour of the day at each household in which the largest volume of water is used.

The national database revealed that residential water use tends to be greater on weekends than on week days. For this reason, weekends were used to identify the peak hour of water use. For a given fixture group, estimates of the peak hour probability of use will vary from household to household, even after classifying by number of occupants. A weighting scheme, based on the duration of the observation window at each home, was used to combine the collection of computed p-values into a single representative estimate for each fixture group. The resulting p-values, rounded off to the nearest 0.005, are listed in Table 5. While these p-values are derived exclusively from water use data measured at single-family homes, they are assumed to also apply to buildings with multiple residential units.

Table 5 Recommended probability of fixture use (*p*) and fixture flow rate (*q*)

Fixture	Design P value (%)	Maximum Recommended Design Flow Rate (gpm)
Bar Sink	2.0	1.5
Bathtub	1.0	5.5
Bidet	1.0	2.0
Clothes Washer	5.5	3.5
Combination Bath/Shower	5.5	5.5
Dishwasher	0.5	1.3
Kitchen Faucet	2.0	2.2
Laundry Faucet	2.0	2.0
Lavatory Faucet	2.0	1.5
Shower, per head	4.5	2.0
Water Closet, 1.28 GPF Gravity Tank	1.0	3.0

3.2 *q-values*

Efficient and ultra-efficient fixture flow rate percentiles were queried from the database as shown in Figure 3. All the recommended fixture flow rates in Table 5 are above the mean (greater than the 75th percentile) for efficient fixtures except for the shower. The efficient shower percentiles reflect the EPACT92 maximum flow rate requirement at 2.2 gpm. The shower flow rate recommendation is based on the EPA WaterSense Specification for Shower Heads and is above the mean for ultra-efficient showers.

The flow rate recommendations for both the lavatory faucet and kitchen faucet are well above the mean for all faucets within the database, exceeding the 90th percentile. This is because the recommendation was based upon the EPA WaterSense High-Efficiency Lavatory Faucet Specification and the 2015 IAPMO Green Plumbing and Mechanical Code Supplement (GPMCS) for kitchen faucets. The GPMCS allows a kitchen faucet flow rate at a temporary maximum 2.2gpm if it defaults back to 1.8gpm. Therefore, the higher flow rate is recommended.

Some fixtures in Table 5 are not included in the database. The bar sink faucet has a comparable flow rate with the lavatory faucet. A residential laundry faucet with an aerator can have a flow rate of 1.5 gpm or 2.0 gpm, the higher flow rate being recommended. Similarly, the bidet faucet flow rate was recommended at 2.0 gpm.

The combination bath/shower has two water outlets that are mutually exclusive. Water will flow either through the tub spout or the shower head from the same fixture fitting. The recommended design flow rate for this fixture fitting is based upon the flow rate for the tub spout and is the same as the bathtub flow rate.

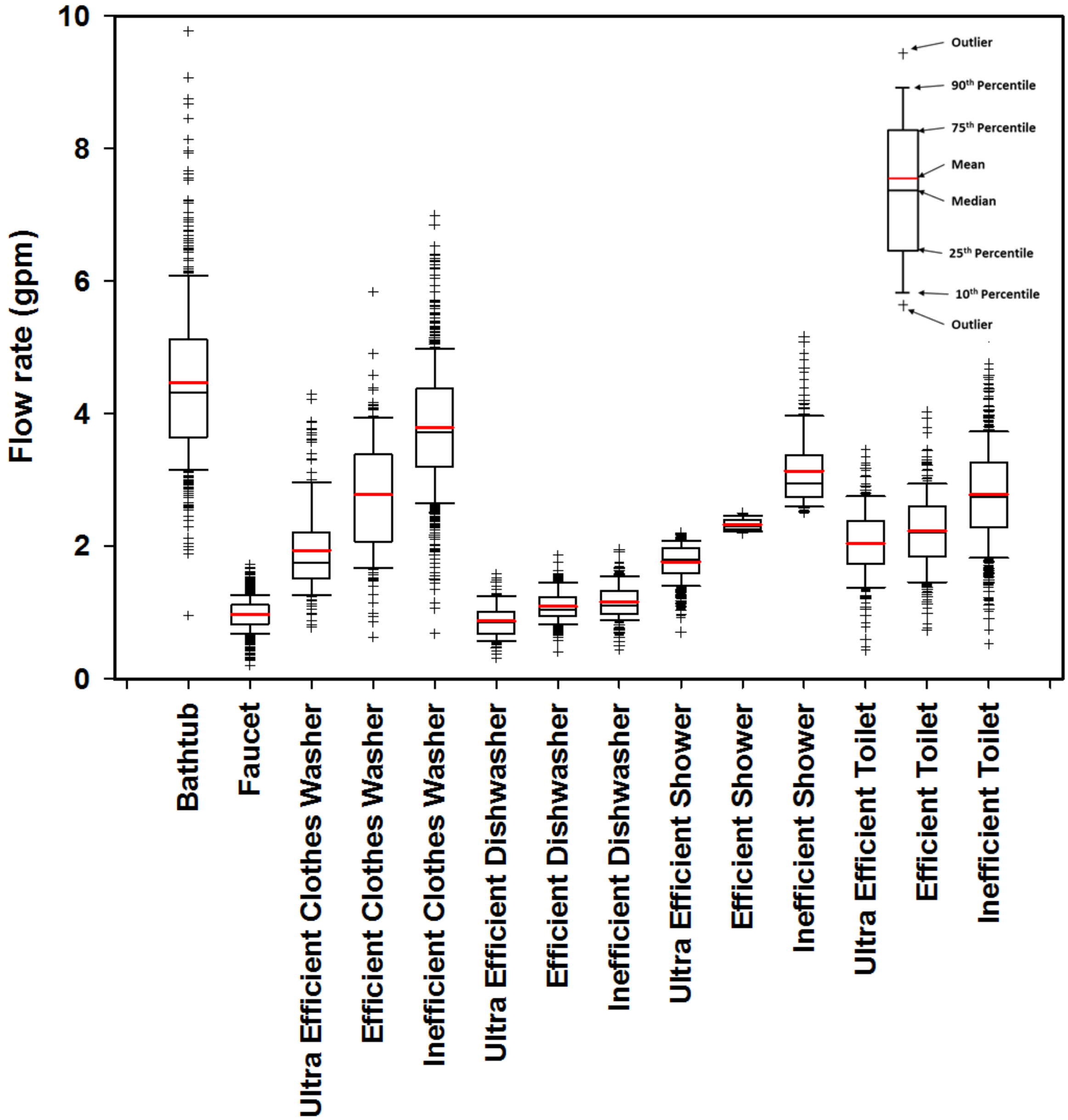

Figure 3: Box and whisker plot of flow rate for the different efficiency level of fixtures.

4.0 Estimating Design Flow

4.1 *Hunter's Method*

Roy Hunter (1940) demonstrated that the use of water fixtures in a building can be described with the binomial probability distribution. Given a group of *n* identical fixtures each with probability *p* of being used, Hunter showed the probability of having exactly *x* fixtures operating simultaneously out of *n* total fixtures has a binomial mass function,

$$\Pr\left[x\ busy\ \ fixtures\,\middle|\,n, p\right] = \binom{n}{x}\left(p\right)^{x}\left(1-p\right)^{n-x} \qquad x = 0,1,\ldots,n \qquad [1]$$

Most buildings have an assortment of fixtures. Each fixture group has their own unique values for *n* and *p* and, hence, their own distinct version of Equation [1]. Hunter used the 99th percentile from Equation [1] as the design standard for estimating peak demand. He recognized, however, that when estimating the load on the building it was not legitimate to simply add the 99th percentile from each fixture group. In a clever move, Hunter introduced *fixture units* to merge the 99th percentile curves for each fixture group into a single design curve giving the 99th percentile of peak demand at a building based on the total fixture units. The final result, called Hunter's Curve, is the theoretical basis for plumbing codes around the world (IAPMO, 2015).

4.2 *Wistort's Method*

Robert Wistort (1994) proposed using the normal approximation for the binomial distribution to estimate directly peak loads on plumbing system. Similar to Hunter's approach, the number of busy fixtures *x* is considered to be a random variable with a binomial distribution having a mean E[*x*] = *np* and variance Var[*x*] = *np*(1-*p*). From the normal approximation, the estimate of the 99th percentile of the demand in a building with *K* different fixture groups is,

$$Q_{0.99} = \sum_{k=1}^{K} n_k\, p_k q_k + \left(z_{0.99}\right)\sqrt{\sum_{k=1}^{K} n_k\, p_k\left(1-p_k\right)q_k^2} \qquad [2]$$

In this expression, n_k is the total number of fixtures belonging to fixture type *k*, p_k is the probability that a single fixture in fixture type *k* is operating, q_k is the flow rate at the busy fixture type *k* and $Z_{0.99}$ is the 99th percentile of the standard normal distribution. The chief advantage of Wistort's method is that it avoids the need for fixture units and is readily extended to other types of fixtures, provided suitable values for *p* and *q* are available.

The normal approximation used in Wistort's method works best when the dimensionless term $H\left(n, p\right) = \sum_{k=1}^{K} n_k\, p_k \geq 5$. This term is called the *Hunter Number* and it represents the expected number of simultaneous busy fixtures in the building during the peak period. In the context of residential plumbing, p_k values tend to be small (average $p \approx 0.03$) so the total number of fixtures must be relatively large (e.g., $\Sigma n \geq 150$) to satisfy the condition $H(n,p) \geq 5$. Consequently, Wistort's method will be suitable for estimating demands at buildings with many residential units, but it is not appropriate for single family homes.

4.3 *Modified Wistort Method*

Single family homes have few occupants and few fixtures. In these cases, idle fixtures are the norm even during the period of peak water use. In a building with K different and independent fixture groups, the probability, P_0, that all fixtures are idle (i.e., zero demand) is,

$$P_0 = \prod_{k=1}^{K} \left(1 - p_k\right)^{n_k} \approx \exp\left[-H\left(n, p\right)\right] \qquad [3]$$

For example, in a 2.5 bath home a typical value for the *Hunter Number* is $H(n,p) = 0.30$. Equation [3] gives $P_0 \approx 0.74$. This implies that the home draws water about 26 percent of the time during the peak demand period, otherwise the entire home is idle. With the conventional binomial distribution, the high probability of idle fixtures in a single family home exerts a strong "downward pull" on the mean number of busy fixtures which, in turn, leads to a significant low bias in the estimated peak flow.

Plumbing systems are not designed for "zero flow" and so this condition should not influence the size of a plumbing system. The task group, therefore, proposes a *zero-truncated binomial distribution* (ZTBD) to describe the conditional probability distribution of busy fixtures in any building, including single family homes. Assuming that a normal approximation can be used to describe the upper tail of the ZTBD, the expression for the 99th percentile of the demand in a building with K different fixture groups is,

$$Q_{0.99} = \frac{1}{1-P_0}\left[\sum_{k=1}^{K} n_k p_k q_k + \left[\left(1+P_0\right) z_{0.99}\right]\sqrt{\left[\left(1-P_0\right)\sum_{k=1}^{K} n_k p_k \left(1-p_k\right) q_k^2\right] - P_0\left(\sum_{k=1}^{K} n_k p_k q_k\right)^2}\,\right] \qquad [4]$$

When $H\left(n, p\right) > 5$, $P_0 \to 0$ and Equation [4] reduces to Equation [2]. For this reason, the ZTBD approach is called the "Modified Wistort Method" (MWM). In practice, the transition from Equation [4] to [2] typically requires at least 100-150 fixtures in the building. Results of Monte Carlo computer runs by the task group to simulate indoor residential water use indicate that MWM works well when the Hunter Number $H\left(n, p\right) \geq 1.25$. A nice feature of MWM is that when n=1 and k=1 (last fixture on the water supply line in the building) Equation [4] simplifies to $Q_{0.99} = q$. The design flow is simply the nominal demand of the final fixture.

4.4 *Exhaustive Enumeration*

Hunter and Wistort focused on the 99th percentile of the demand expected during the peak period in a building. With today's computational tools it is possible to numerically generate the entire probability distribution of the water demands at any point and any time in a building. "Exhaustive Enumeration" involves identifying and ranking all possible demand events for a given premise plumbing configuration. To illustrate, consider the peak period in a kitchen/laundry space having a total of n=4 independently operated fixtures listed in Table 6.

Table 6 Parameters required to estimate peak demand.

Symbol	Table 7 Color	Fixture	n_k	p_k	q_k (gpm)	Rank
CW	(yellow)	Clothes washer	1	0.055	3.5	1
DW	(green)	Dishwasher	1	0.005	1.3	4
KF	(blue)	Kitchen faucet	1	0.020	2.2	2
LF	(orange)	Laundry faucet	1	0.020	2.0	3

The *Hunter Number* for the 4 fixtures in Table 6 is $H(n,p) = \sum n_k p_k = 0.10$. Equation [3] gives the corresponding probability of zero demand as P_0 = exp[-0.10] = 0.904. There are 2^4 = 16 mutually exclusive combinations of fixture use as summarized in Table 7. Column 10 of case 1 gives the probability of zero demand as 0.903, in agreement with the result from Equation [3]. The estimated 99th percentile of the conditional busy-time demand is $Q_{0.99}$ = 5.5 gpm, highlighted in columns [12] and [14].

When the probability of zero demand is high, there can be significant differences in the cumulative probability of the household demands depending on whether an unconditional "total-time" or a conditional "busy-time' view is adopted. This is illustrated in Figure 4, which shows the probability distribution of total-time demands in **blue** and busy-time demands in **red** for the four-fixture example given in Tables 6 and 7. The 90 percent spike at zero demand dominates the **blue** total-time plot. The median (50th percentile) demand is $Q_{0.50}$ = 0 gpm and the 99th percentile demand is $Q_{0.99}$ = 3.5 gpm. In contrast, the **red** busy-time plot excludes zero demand by definition. The busy-time plot has median demand of 3.5 gpm and a 99th percentile demand of $Q_{0.99}$ = 5.5 gpm, as confirmed in Columns [12] and [14] of Table 7.

Table 7 Exhaustive enumeration of 16 mutually exclusive demand outcomes.

[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]
Case	CW	DW	KF	LF	P_{CW}	P_{DW}	P_{KF}	P_{LF}	Q (gpm)	T.T. Probability	Q Ranked	B.T. Probability	B.T. CDF
1	○	○	○	○	0.945	0.995	0.980	0.980	0.0	0.9030401	0.0		0.000
2	●	○	○	○	0.055	0.995	0.980	0.980	3.5	0.0525579	1.3	0.046802	0.047
3	○	●	○	○	0.945	0.005	0.980	0.980	1.3	0.0045379	2.0	0.190072	0.237
4	○	○	●	○	0.945	0.995	0.020	0.980	2.2	0.0184294	2.2	0.190072	0.427
5	○	○	○	●	0.945	0.995	0.980	0.020	2.0	0.0184294	3.3	0.000955	0.428
6	●	●	○	○	0.055	0.005	0.980	0.980	4.8	0.0002641	3.5	0.542058	0.970
7	●	○	●	○	0.055	0.995	0.020	0.980	5.7	0.0010726	3.5	0.000955	0.971
8	●	○	○	●	0.055	0.995	0.980	0.020	5.5	0.0010726	4.2	0.003879	0.975
9	○	●	●	○	0.945	0.005	0.020	0.980	3.5	0.0000926	4.8	0.002724	0.978
10	○	●	○	●	0.945	0.005	0.980	0.020	3.3	0.0000926	**5.5**	0.011062	**0.989**
11	○	○	●	●	0.945	0.995	0.020	0.020	4.2	0.0003761	**5.5**	0.000019	**0.989**
12	●	●	●	○	0.055	0.005	0.020	0.980	7.0	0.0000054	5.7	0.011062	1.000
13	●	●	○	●	0.055	0.005	0.980	0.020	6.8	0.0000054	6.8	0.000056	1.000
14	●	○	●	●	0.055	0.995	0.020	0.020	7.7	0.0000219	7.0	0.000056	1.000
15	○	●	●	●	0.945	0.005	0.020	0.020	5.5	0.0000019	7.7	0.000226	1.000
16	●	●	●	●	0.055	0.005	0.020	0.020	9.0	0.0000001	9.0	0.000001	1.000
									Sum	1.0000000	Sum	1.000000	

Key for Table 7

Column [1]	16 mutually exclusive collectively exhaustive demand outcomes from 4 fixtures
Cols [2] - [5]	● indicates fixture is busy; ○ indicates fixture is idle
Cols [6] - [9]	Shaded values are Prob[fixture is busy]; unshaded values are Prob[fixture is idle]
Column [10]	Demand (gpm) for each outcome, ranging from 0 to 9 gpm. For example, Case 7 represents simultaneous use of the clothes washer and the kitchen faucet. The total demand is the sum of both draws, 3.5 gpm + 2.2 gpm = 5.7 gpm.
Column [11]	"Total-time" probability of an outcome, found as the product of Cols [6] thru [9]; The zero demand condition of Case 1 dominates the total-time picture.
Column [12]	Demands of Col [10] ranked from minimum to maximum.
Column [13]	"Busy-Time" conditional probability, found as Col [11] / [1-P_0], for the corresponding ranked demand in Col [12]; Zero demand condition is excluded from busy-time.
Column [14]	Cumulative busy-time conditional probability, from running sum of Col [13]; The 99th percentile is reached at Cases 10 and 11 with a demand of $Q_{0.99}$ = 5.5 gpm.

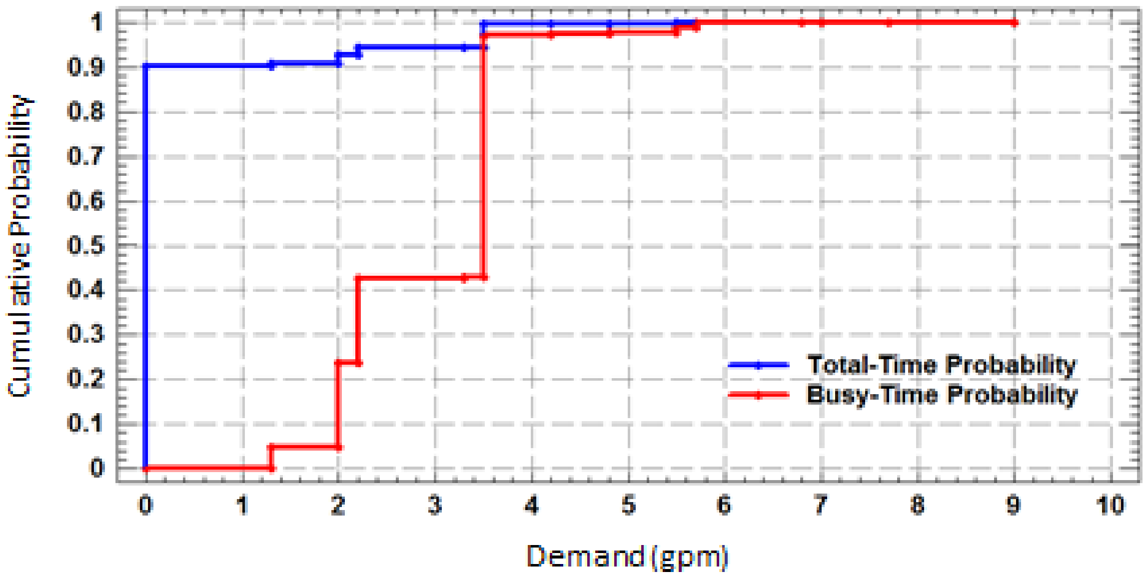

Figure 4 Cumulative probability plots for the 4-fixture example in Tables 6 and 7.

While simple in principle, exhaustive enumeration may not yet always be practical. Due to combinatorial explosion, the size of the problem grows geometrically. The small example in Table 6 with four independent fixtures generated $2^4 = 16$ possible demand outcomes. A typical single family home with 12 independent fixtures will generate $2^{12} = 4{,}096$ demand outcomes. So while enumeration is helpful to visualize the jagged probability distribution of fixture demands, it may be restricted to scenarios where the total fixture count *n* is relatively small. Additional examples featuring exhaustive enumeration can be found in Buchberger *et al* (2012) and Omaghomi and Buchberger (2014).

4.5 *q1+q3 Method*

During development of the exhaustive enumeration approach, it was observed that certain combinations of fixtures consistently tended to strike near the 99th percentile design demand, especially along branch lines at the household level. This led to the "q1+q3" method which works as follows: Using recommended fixture demand values, rank all fixtures along a branch line in descending order. For instance, the fixture with the largest demand receives rank of 1 (q1), the fixture with the second largest demand receives rank of 2 (q2), and so on until all fixtures on a designated branch are ranked. The q1+q3 method simply adds the demands for the rank 1 and the rank 3 fixtures to obtain an expedient and reasonably good estimate of the 99th percentile demand, often identical to the value generated by a full exhaustive enumeration.

To demonstrate, consider the four fixtures in Table 6. Rank 1 goes to the clothes washer with its demand of **q1=3.5 gpm**; Rank 2 goes to the kitchen faucet with its demand of q2=2.2 gpm; Rank 3 goes to the laundry faucet with its demand of **q3=2.0 gpm**; and Rank 4 goes to the dishwasher with its demand of q4=1.3 gpm. According to the q1+q3 method, the design demand is 3.5 gpm + 2.0 gpm = 5.5 gpm, produced by simultaneous use of the clothes washer and the laundry faucet. This agrees with the result shown earlier for exhaustive enumeration in Table 7 and the busy-time plot in Figure 4. Excellent results with the q1+q3 method have been obtained for *n* approaching 10 or 15 fixtures, provided the overall average *p*-value is not too high.

If a branch line has two or more identical fixtures, each fixture receives a unique rank. Suppose, for example, that a second clothes washer were added to Table 6 and the other fixtures remain in place. Rank 1 would stay with clothes washer #1, while Rank 2 would now go to clothes washer #2 and Rank 3 would go to the kitchen faucet. The design demand from the q1+q3 method now increases slightly to 3.5 gpm + 2.2 gpm = 5.7 gpm, ostensibly produced by simultaneous use of the clothes washer and the kitchen faucet. If a branch has *n*=2 fixtures, then q3 = 0 and the design demand is q1, that is, the greater of the two fixture demands. Finally, if a branch is supplying only one fixture, then the design demand is q1.

4.6 *Method Summary*

Various methods for estimating peak demands in buildings were examined, developed and tested. The problem is challenging because the solution strategy changes with the spatial scale of the plumbing system. On a large scale [i.e., total fixture count, $n > 200$], individual fixtures do not appreciably affect the performance of the water supply system. As a consequence, solutions like the Wistort method using well-established continuous probability distributions can be applied readily to estimate demands. On a small scale [$n < 20$], individual fixtures exert a significant impact on system behavior. At this level, solutions like exhaustive enumeration or q1+q3 are needed as they account for discrete fixtures in premise plumbing.

Table 8 summarizes the four main methods that the Task Group investigated for estimating peak demands in residential buildings. The applicability of each method corresponds generally to a region [A, B, C, D] in the *n-p* plane of Figure 5. These regions are delineated by diagonal lines that represent a constant *Hunter Number* (defined on page 8).

Table 8 Methods to estimate peak demands in residential buildings with efficient fixtures.

Region	Spatial Scale	Range for $H(n,p)$	Method
A	Small	$0 \leq H(n,p) \leq 0.25$	Exhaustive Enumeration; q1+q3
B	Small to Intermediate	$0.25 \leq H(n,p) \leq 1.25$	Exhaustive Enumeration
C	Intermediate to Large	$1.25 \leq H(n,p) \leq 5.00$	Modified Wistort Method
D	Large	$5.00 \leq H(n,p)$	Wistort Method

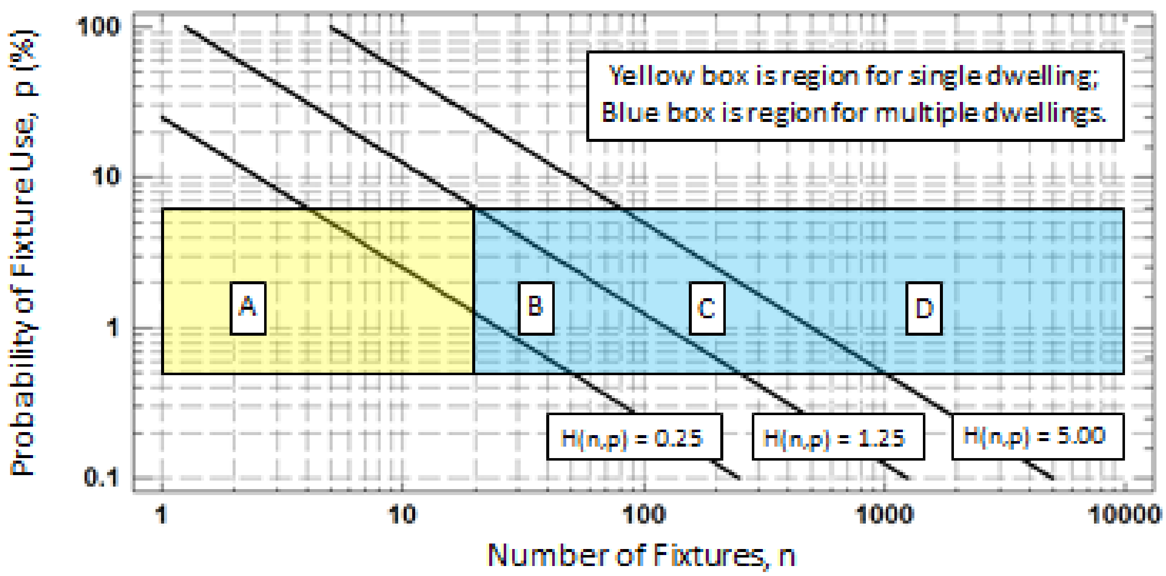

Figure 5 Conditions of interest for periods of peak demand in residential plumbing tend to occur in the yellow box (single dwelling) and the blue box (multi-family dwellings)

5.0 Water Demand Calculator [WDC]

Aside from the q1+q3 method for single family households, the other three approaches for estimating peak water demand in a building are cumbersome to use, even when the designer knows all the required input parameters (i.e., *n, p, q*) for each fixture group. To encourage proper use and promote uniform application of these proposed new approaches for estimating peak indoor demands, a Water Demand Calculator (WDC) has been developed. The WDC will be provided to the user as a downloadable Microsoft Office Excel spreadsheet.

A screen shot of the template for the WDC is shown in Figure 6. The WDC has white-shaded cells and blue-shaded cells. The values in the blue cells are derived from a national survey of indoor water use at homes with efficient fixtures. These values cannot be changed. The white-shaded cells accept input from the designer. For instance, fixture counts from the four-fixture example in Table 6 are entered in Column [B]; the corresponding recommended fixture flow rates listed in Table 5 are already provided in Column [D]. The flow rates in Column [D] may be reduced only if the manufacturer specifies a lower flow rate for the fixture. Column [E] establishes the upper limits for the flow rates entered into Column [D]. Clicking the ***Run Water Demand Calculator*** button gives 5.5 gpm as the estimated indoor water demand. This result agrees with the exhaustive enumeration result for the 99th percentile in Table 7 and appears below in the green box of the WDC in Figure 6.

	[A] FIXTURE	[B] ENTER NUMBER OF FIXTURES	[C] PROBABILITY OF USE (%)	[D] ENTER FIXTURE FLOW RATE (GPM)	[E] MAXIMUM RECOMMENDED FIXTURE FLOW RATE (GPM)
1	Bar Sink	0	2.0	1.5	1.5
2	Bathtub	0	1.0	5.5	5.5
3	Bidet	0	1.0	2.0	2.0
4	Clothes Washer	1	5.5	3.5	3.5
5	Combination Bath/Shower	0	5.5	5.5	5.5
6	Dishwasher	1	0.5	1.3	1.3
7	Kitchen Faucet	1	2.0	2.2	2.2
8	Laundry Faucet	1	2.0	2.0	2.0
9	Lavatory Faucet	0	2.0	1.5	1.5
10	Shower, per head	0	4.5	2.0	2.0
11	Water Closet, 1.28 GPF Gravity Tank	0	1.0	3.0	3.0
12	Other Fixture 1	0	0.0	0.0	6.0
13	Other Fixture 2	0	0.0	0.0	6.0
14	Other Fixture 3	0	0.0	0.0	6.0

Total Number of Fixtures 4

99th PERCENTILE DEMAND FLOW = 5.5 GPM

RESET

RUN WATER DEMAND CALCULATOR

Figure 6 Screen shot of the input/output template for the Water Demand Calculator.

6.0 Example with Single and Multi-Family Dwelling

Several examples using the WDC are presented in Tables 9 and 10 for estimating peak cold water and hot water demand, respectively. Both Tables include four scenarios ranging from a single family dwelling [region A in Figure 5] to a multi-family building with 27 identical units [region D in Figure 5]. The estimated 99th percentile of the cold water demand exceeds the 99th percentile for the hot water demand by an average of about 4 percent. The slightly higher demand for cold water is due to the fact that there are 10 cold water fixtures, but only 8 hot water fixtures in this example. The peak demands for the cold water case would be used to size the main cold water only branch. For comparison, q1+q3 gives a design demand of 9.0 gpm for the cold water branch. The WDC checks q1+q3 against exhaustive enumeration and selects the greater result (in this case 11.0 gpm for both the cold water branch and the hot water branch).

Table 9 Example **Cold Water** Demand for Single and Multi-Family Units from the WDC.

Fixture	*p*-value	*q*-value (gpm)	Number of Each Fixture, *n*			
			1 Unit	3 Units	9 Units	27 Units
Clothes washer	0.055	3.5	1	3	9	27
Combo tub/shower	0.055	5.5	2	6	18	54
Dishwasher	0.005	1.3	0	0	0	0
Kitchen faucet	0.020	2.2	1	3	9	27
Lavatory faucet	0.020	1.5	3	9	27	81
Toilet	0.010	3.0	3	9	27	81
Total **Cold Fixtures**			10	30	90	270
Region on Figure 5			A	B	C	D
Dimensionless Hunter Number, $H(n,p)$			0.275	0.825	2.475	7.425
Prob[zero demand during peak period]			0.760	0.438	0.084	0.001
99th Percentile Demand $Q_{0.99}$ (gpm)			**11.0**	**15.5**	**24.6**	**52.6**

Table 10 Example **Hot Water** Demand for Single and Multi-Family Units from the WDC

Fixture	*p*-value	*q*-value (gpm)	Number of Each Fixture, *n*			
			1 Unit	3 Units	9 Units	27 Units
Clothes washer	0.055	3.5	1	3	9	27
Combo tub/shower	0.055	5.5	2	6	18	54
Dishwasher	0.005	1.3	1	3	9	27
Kitchen faucet	0.020	2.2	1	3	9	27
Lavatory faucet	0.020	1.5	3	9	27	81
Toilet	0.010	3.0	0	0	0	0
Total **Hot Fixtures**			8	24	72	216
Region on Figure 5			A	B	C	D
Dimensionless Hunter Number, $H(n,p)$			0.25	0.75	2.25	6.75
Prob[zero demand during peak period]			0.779	0.472	0.105	0.001
99th Percentile Demand $Q_{0.99}$ (gpm)			**11.0**	**14.5**	**23.7**	**49.6**

7.0 Conclusions and Recommendations

The computational methods for estimating water supply demand for single and multi-family dwellings identified in this report and coded into the Water Demand Calculator are offered as an improved method to avoid over-design resulting from Hunter's Curve as the current method used in U.S. plumbing codes.

The accuracy of the methods is dependent upon the key parameters of fixture use probability and fixture flow rate. The end use of water data used in this report is from the largest available US residential end use of water survey (REUWS) provided by Aquacraft, Inc. The data requested from Aquacraft's REUWS specified homes having low water consumption fixtures. Although this trimmed a large portion of available data that did not contain efficient fixtures, the remaining sample size was more than adequate to conduct statistical analysis. The fixture *p*-values and *q*-values are sound based upon the data available. This effort far exceeds the sparse data used in the development of the Hunter method which consisted of a few hotels and government offices.

The modified Wistort method holds promise as a tractable analytical expression to directly estimate the design discharge for a wide spectrum of buildings, ranging from small private dwellings to large public facilities. With proper scaling, the modified Wistort equation can be formulated as a universal dimensionless design expression for determining the peak water demand expected for buildings in the residential, commercial, institutional and other sectors.

A key advantage of the Wistort approach is that it does not rely on mysterious fixture units and it is not calibrated to any particular fixture type. Hence, the dimensionless formulation will remain valid even as water use habits change and fixture types evolve in the future. The modified Wistort method is easily programmed on an electronic spreadsheet, can be offered as a convenient "app" to engineers and inspectors, and is readily incorporated into emerging digital tools (i.e., BIM) of the Architectural-Engineering-Construction industry.

Some specific areas for potential future research include:

- Implement a broad national field program to measure instantaneous peak indoor water demands in the residential sector and at other end users (commercial, institutional, etc). These data are needed to calibrate and validate the proposed modified Wistort method for estimating peak indoor water demand.
- Analyze data from the national monitoring program to extend and refine estimates of *p* and *q* for a wide range of end uses (residential, commercial, institutional, public, etc).
- Establish a comprehensive "cloud bank" to serve as an on-line digital repository for *p* and *q* values for every fixture that is in use today or becomes available in the future.
- Investigate the use of other probability distributions (besides the standard normal distribution) as potential candidates to better represent the upper tail of the zero truncated binomial distribution for use in cases where the fixture count *n* is small.
- In anticipation of continuing advances in computational speed and data storage, develop better algorithms to implement exhaustive enumeration so that this basic method can be extended estimate peak demands for large multi-family dwellings and eventually span all the regions [A, B, C, D] illustrated in Figure 5.

8.0 References

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